NAG Toolbox for MATLAB

f07fn

1 Purpose

f07fn computes the solution to a complex system of linear equations

$$AX = B,$$

where A is an n by n Hermitian positive-definite matrix and X and B are n by r matrices.

2 Syntax

```
[a, b, info] = f07fn(uplo, a, b, 'n', n, 'nrhs_p', nrhs_p)
```

3 Description

f07fn uses the Cholesky decomposition to factor A as $A = U^{H}U$ if **uplo** = 'U' or $A = LL^{H}$ if **uplo** = 'L', where U is an upper triangular matrix and L is a lower triangular matrix. The factored form of A is then used to solve the system of equations $AX = B$.

4 References

Anderson E, Bai Z, Bischof C, Blackford S, Demmel J, Dongarra J J, Du Croz J J, Greenbaum A, Hammarling S, McKenney A and Sorensen D 1999 *LAPACK Users' Guide* (3rd Edition) SIAM, Philadelphia URL: http://www.netlib.org/lapack/lug

Golub G H and Van Loan C F 1996 *Matrix Computations* (3rd Edition) Johns Hopkins University Press, Baltimore

5 Parameters

5.1 Compulsory Input Parameters

1: **uplo – string**

If **uplo** = 'U', the upper triangle of A is stored.

If **uplo** = 'L', the lower triangle of A is stored.

Constraint: **uplo** = 'U' or 'L'.

2: **a(lda,*) – complex array**

The first dimension of the array **a** must be at least $\max(1,\mathbf{n})$

The second dimension of the array must be at least $\max(1,\mathbf{n})$

The n by n Hermitian matrix A.

If **uplo** = 'U', the upper triangular part of A must be stored and the elements of the array below the diagonal are not referenced.

If **uplo** = 'L', the lower triangular part of A must be stored and the elements of the array above the diagonal are not referenced.

3: **b(ldb,*) – complex array**

The first dimension of the array **b** must be at least $\max(1,\mathbf{n})$

The second dimension of the array must be at least $\max(1,\mathbf{nrhs\_p})$

Note: To solve the equations $Ax = b$, where b is a single right-hand side, **b** may be supplied as a one-dimensional array with length **ldb** $= \max(1, \mathbf{n})$.

The n by r right-hand side matrix B.

5.2 Optional Input Parameters

1: **n – int32 scalar**

Default: The second dimension of the array **a**.

n, the number of linear equations, i.e., the order of the matrix A.

Constraint: $\mathbf{n} \geq 0$.

2: **nrhs_p – int32 scalar**

Default: The second dimension of the array **b**.

r, the number of right-hand sides, i.e., the number of columns of the matrix B.

Constraint: **nrhs_p** ≥ 0.

5.3 Input Parameters Omitted from the MATLAB Interface

lda, ldb

5.4 Output Parameters

1: **a(lda,$*$) – complex array**

The first dimension of the array **a** must be at least $\max(1, \mathbf{n})$

The second dimension of the array must be at least $\max(1, \mathbf{n})$

If **info** $= 0$, the factor U or L from the Cholesky factorization $A = U^{\mathrm{H}}U$ or $A = LL^{\mathrm{H}}$.

2: **b(ldb,$*$) – complex array**

The first dimension of the array **b** must be at least $\max(1, \mathbf{n})$

The second dimension of the array must be at least $\max(1, \mathbf{nrhs\_p})$

Note: To solve the equations $Ax = b$, where b is a single right-hand side, **b** may be supplied as a one-dimensional array with length **ldb** $= \max(1, \mathbf{n})$.

If **info** $= 0$, the n by r solution matrix X.

3: **info – int32 scalar**

info $= 0$ unless the function detects an error (see Section 6).

6 Error Indicators and Warnings

Errors or warnings detected by the function:

info $= -i$

If **info** $= -i$, parameter i had an illegal value on entry. The parameters are numbered as follows:

1: **uplo**, 2: **n**, 3: **nrhs_p**, 4: **a**, 5: **lda**, 6: **b**, 7: **ldb**, 8: **info**.

It is possible that **info** refers to a parameter that is omitted from the MATLAB interface. This usually indicates that an error in one of the other input parameters has caused an incorrect value to be inferred.

info > 0

If **info** $= i$, the leading minor of order i of A is not positive-definite, so the factorization could not be completed, and the solution has not been computed.

7 Accuracy

The computed solution for a single right-hand side, $\hat{x}$, satisfies an equation of the form

$$(A + E)\hat{x} = b,$$

where

$$\|E\|_1 = O(\epsilon)\|A\|_1$$

and ϵ is the ***machine precision***. An approximate error bound for the computed solution is given by

$$\frac{\|\hat{x} - x\|_1}{\|x\|_1} \leq \kappa(A)\frac{\|E\|_1}{\|A\|_1},$$

where $\kappa(A) = \left\|A^{-1}\right\|_1 \|A\|_1$, the condition number of A with respect to the solution of the linear equations. See Section 4.4 of Anderson *et al.* 1999 for further details.

f07fp is a comprehensive LAPACK driver that returns forward and backward error bounds and an estimate of the condition number. Alternatively, f04cd solves $Ax = b$ and returns a forward error bound and condition estimate. f04cd calls f07fn to solve the equations.

8 Further Comments

The total number of floating-point operations is approximately $\frac{4}{3}n^3 + 8n^2r$, where r is the number of right-hand sides.

The real analogue of this function is f07fa.

9 Example

```
uplo = 'Upper';
a  =  [complex(3.23,  +0),  complex(1.51,  -1.92),  complex(1.9,  +0.84),
complex(0.42, +2.5);
      complex(0, 0), complex(3.58, +0), complex(-0.23, +1.11), complex(-
1.18, +1.37);
       complex(0, 0), complex(0, 0), complex(4.09, +0), complex(2.33, -
0.14);
     complex(0, 0), complex(0, 0), complex(0, 0), complex(4.29, +0)];
b = [complex(3.93, -6.14);
     complex(6.17, +9.42);
     complex(-7.17, -21.83);
     complex(1.99, -14.38)];
[aOut, bOut, info] = f07fn(uplo, a, b)
```

```
aOut =
    1.7972                0.8402 - 1.0683i    1.0572 + 0.4674i    0.2337 +
1.3910i
         0               1.3164              -0.4702 - 0.3131i    0.0834 -
0.0368i
         0                    0               1.5604              0.9360 -
0.9900i
        0                  0                  0              0.6603
bOut =
   1.0000 - 1.0000i
  -0.0000 + 3.0000i
  -4.0000 - 5.0000i
   2.0000 + 1.0000i
info =
```

```
0
```